ECE 330: Power Circuits and Electromechanics

Lecture 25

2019-12-04

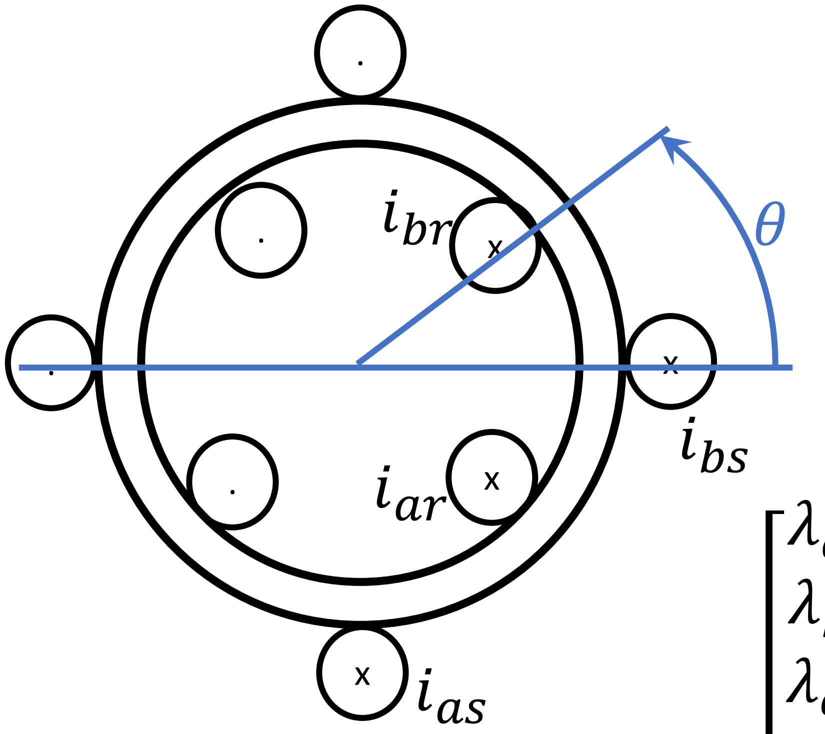
Previously

- Rotating Magnetic Fields
- Synchronous Machines

Today

- Induction Machines
- Equivalent Circuit for Induction Machines

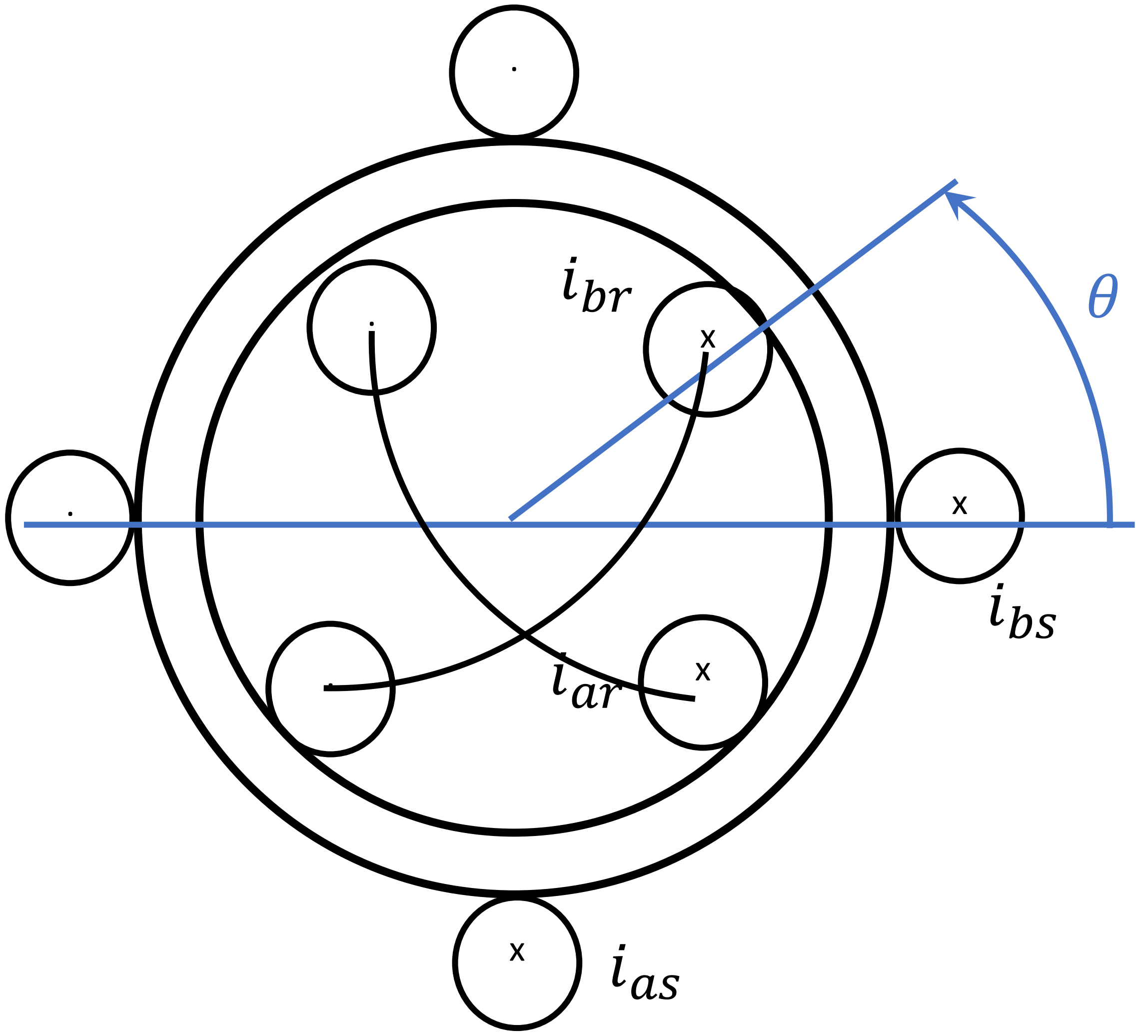
2-Phase Machine



$$\begin{bmatrix}\lambda_{as}\\ \lambda_{bs}\\ \lambda_{ar}\\ \lambda_{br}\end{bmatrix} = \begin{bmatrix} L_s & 0 & M\cos(\theta) & -M\sin(\theta) \\ 0 & L_s & M\sin(\theta) & M\cos(\theta) \\ M\cos(\theta) & M\sin(\theta) & L_r & 0 \\ -M\sin(\theta) & M\cos(\theta) & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{as}\\ i_{bs}\\ i_{ar}\\ i_{br}\end{bmatrix}$$

Same equations for flux linkage as before.

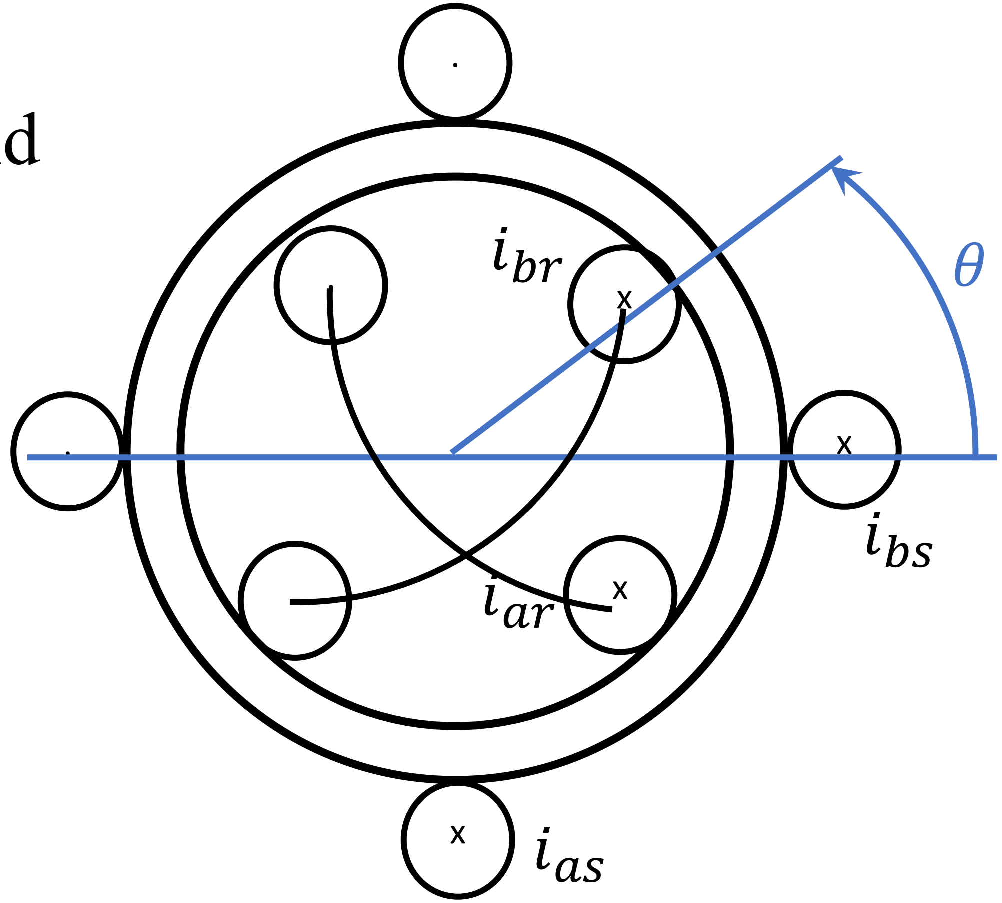
2-Phase Machine (Continued)



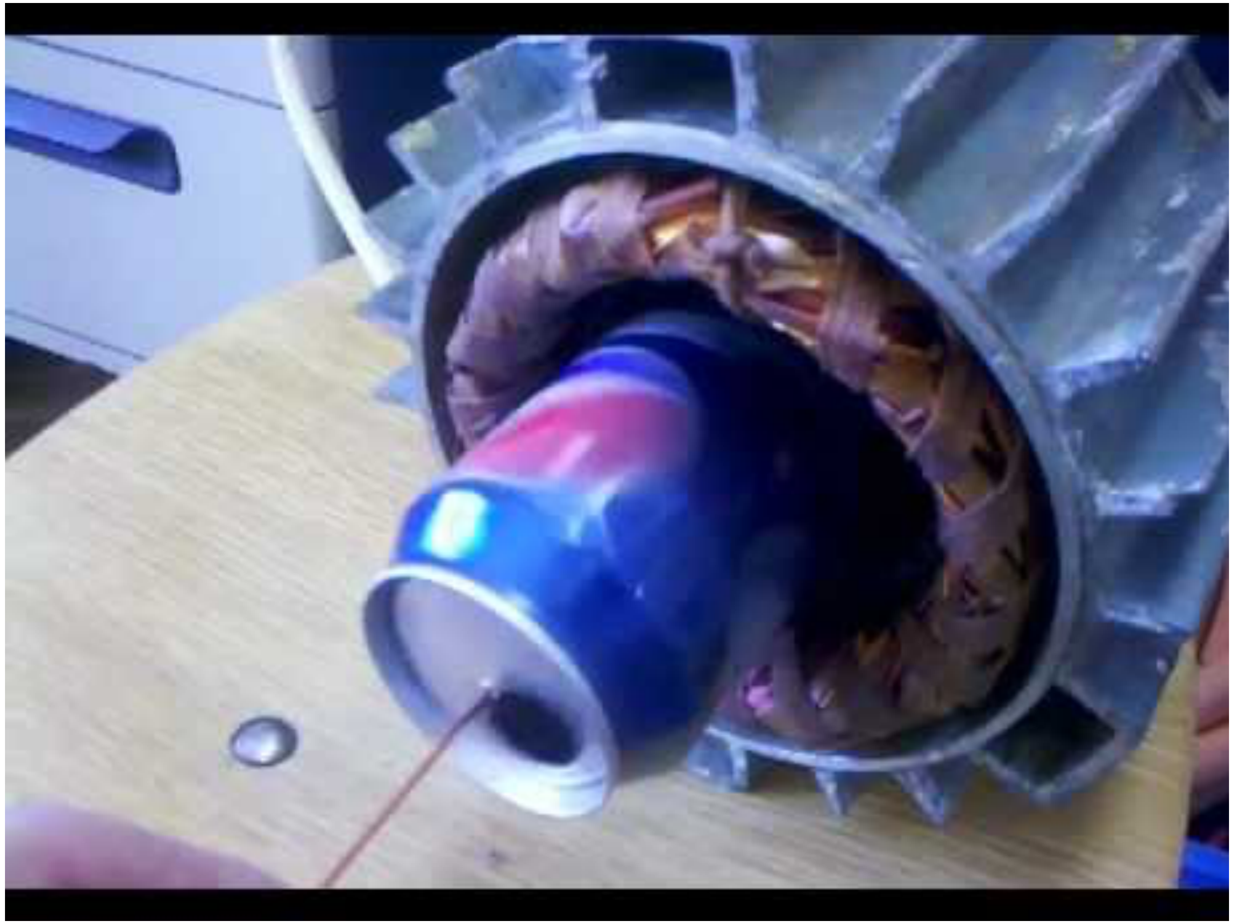
For an induction machine, we short the rotor windings.

2-Phase Machine (Continued)

- Stator still produces rotating magnetic field
- Rotor is now a conductor in a changing magnetic field
  Induced currents in the rotor!
- Induced currents flow in direction to oppose changing magnetic field
  Lenz's law
- Stator and rotor magnetic fields act to produce torque causing rotor to rotate in the same direction as stator field



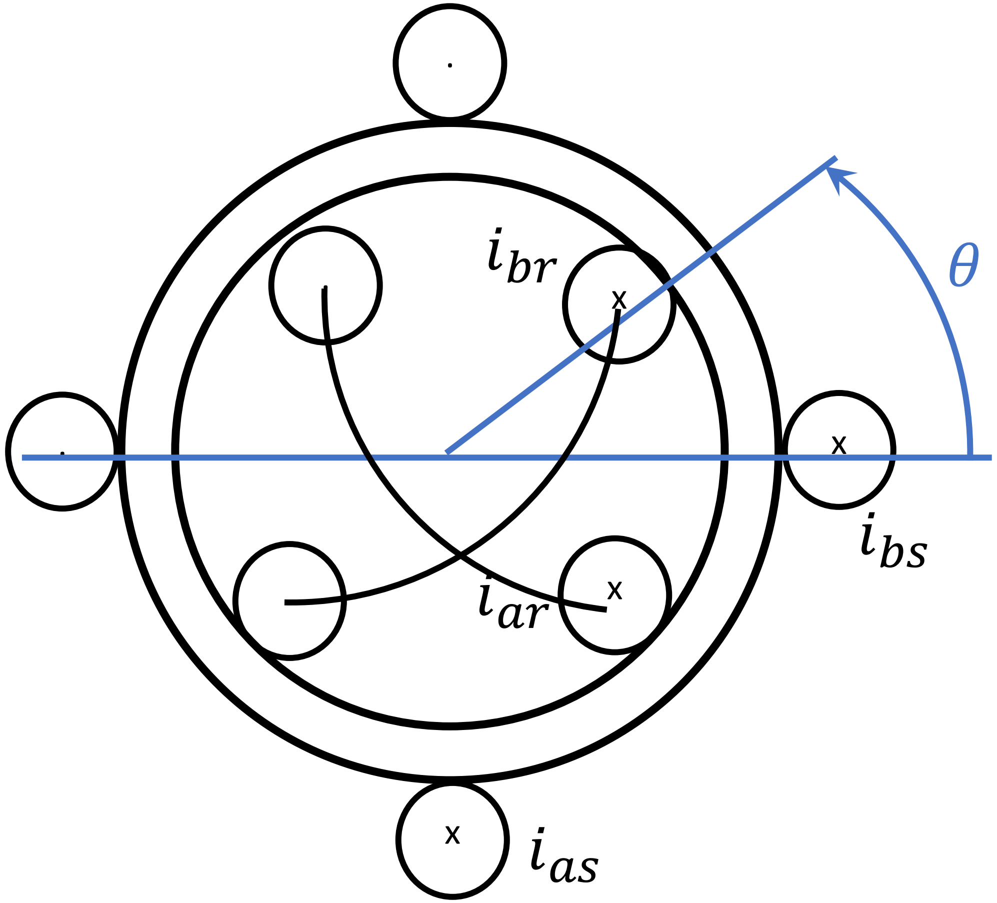
Pop Can Induction Motor



https://www.youtube.com/watch?v=z-oue39E5PA&list=WL&index=19&t=0s

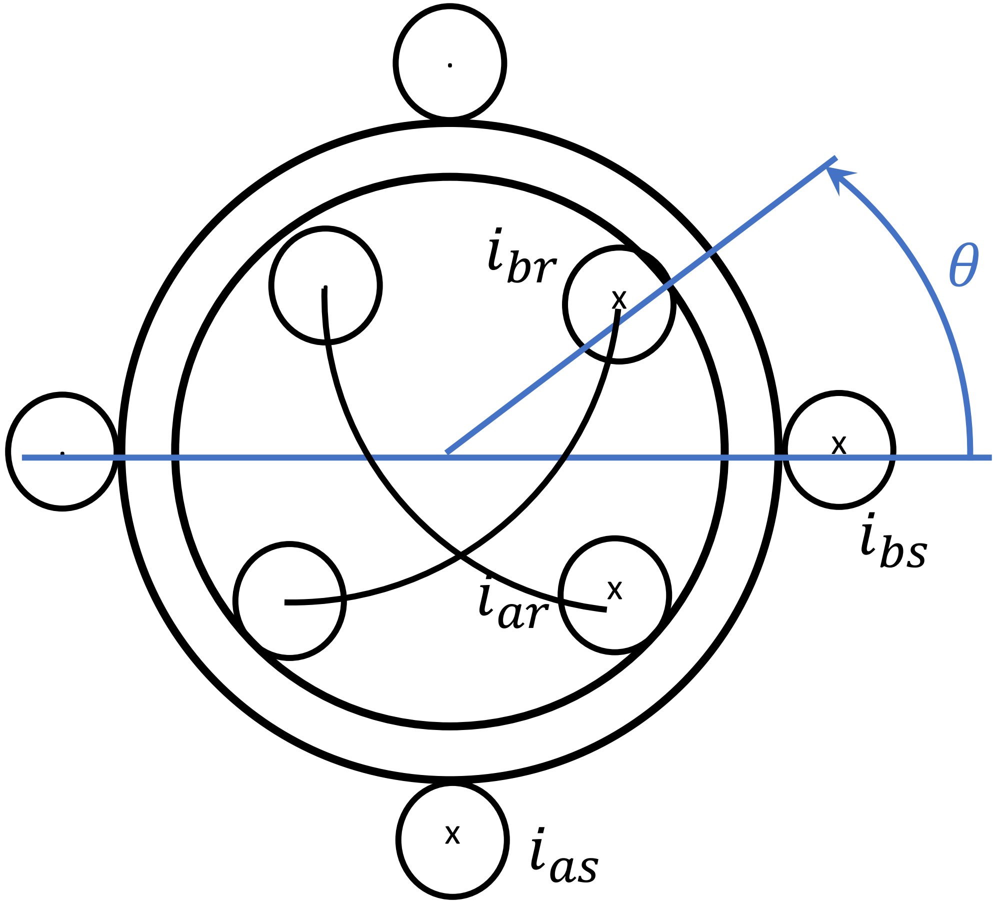
2-Phase Machine (Continued)

- Need stator field to be changing relative to rotor to induce current
- Rotor spins at a speed slightly slower than synchronous speed
- Slip: $s = \frac{\omega_s - \omega_m}{\omega_s}$
- s is usually very small



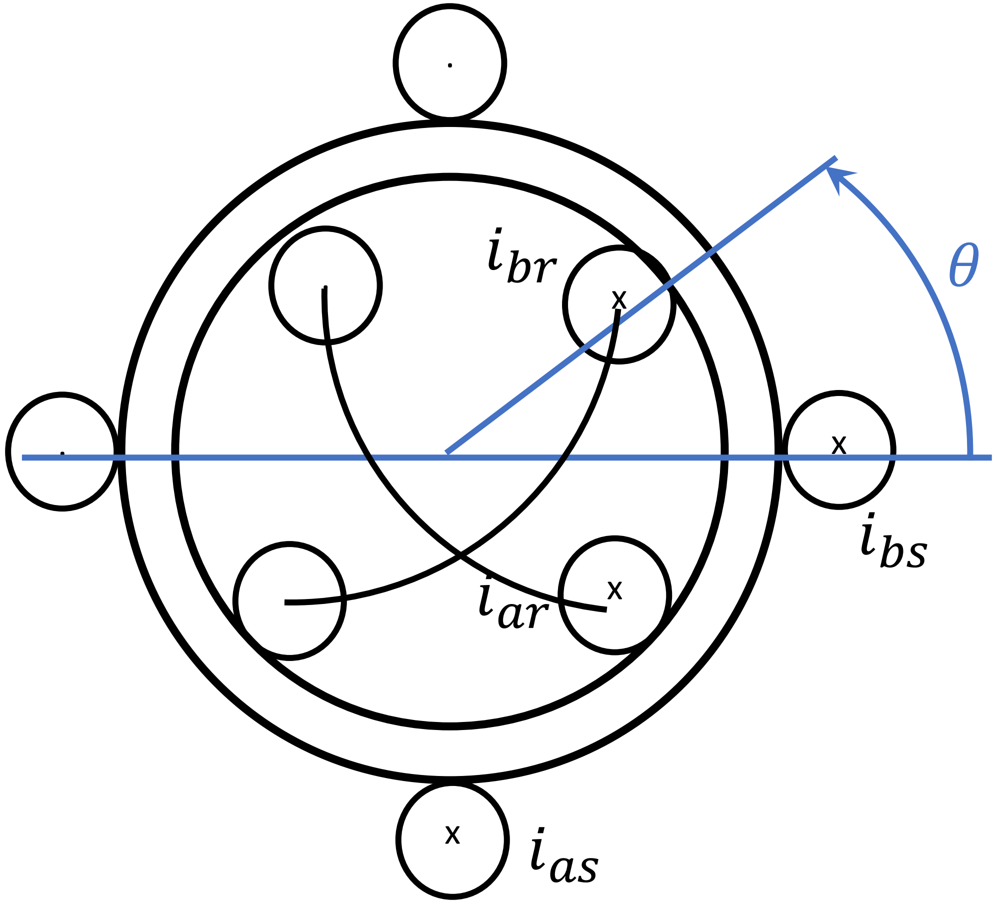
2-Phase Machine (Continued)

- Frequency condition:
  $\omega_m = \omega_s - \omega_r$
- From slip definition:
  $\omega_m = (1 - s)\omega_s$
- Rotor AC frequency:
  $\omega_r = s\omega_s$



Deriving Equivalent Circuit

- Assume
  $i_{as} = I_s \cos(\omega_s t)$
  $i_{bs} = I_s \sin(\omega_s t)$
  $i_{ar} = I_r \cos(s\omega_s t + \beta)$
  $i_{br} = I_r \sin(s\omega_s t + \beta)$
  $\theta = (1 - s)\omega_s + \theta_0$
- $v_{as} = i_{as}R_s + \frac{d\lambda_{as}}{dt}$
- $v_{ar} = i_{as}R_r + \frac{d\lambda_{ar}}{dt}$



Deriving Equivalent Circuit (Continued)

- $V_s \cos(\omega_s t + \theta_s) = R_s I_s \cos(\omega_s t) + \omega_s L_s I_s \cos(\omega_s t + 90°) + \omega_s M I_r \cos(\omega_s t + (\theta_0 + \beta + 90°))$
- $V_r \cos(s\omega_s t + \theta_r) = R_r I_r \cos(s\omega_s t + \beta) + s\omega_s L_r I_r \cos(s\omega_s t + (\beta + 90°)\ ) + s\omega_s M I_r \cos(s\omega_s t + (90° - \theta_0))$

Deriving Equivalent Circuit (Continued)

- Recall: $M = \frac{\mu_0 \pi r l N_s N_r}{2g}$
- Can use this to write: $L_s = L_{ls} + \left(\frac{N_s}{Nr}\right) M, L_r = L_{lr} + \left(\frac{N_r}{N_s}\right) M$
- Can also write rotor values referenced from the stator (similar to transformer circuit)

$$I_r' = I_r \left(\frac{N_r}{N_s}\right), V_r' = V_r \left(\frac{N_s}{N_r}\right)$$

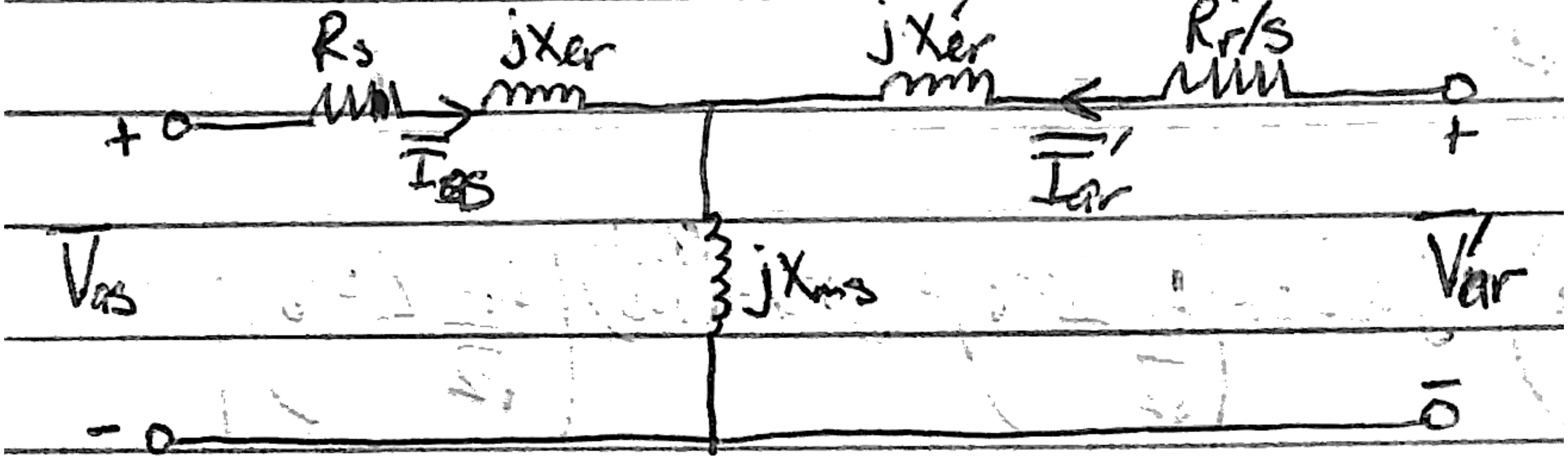
$$R_r' = R_r \left(\frac{N_s}{N_r}\right)^2, L_{lr}' = L_{lr} \left(\frac{N_s}{N_r}\right)^2$$

Deriving Equivalent Circuit (Continued)

As a circuit: $X_{ls} = \omega_s L_{ls}$

$X_{ms} = \omega_s \left(\frac{N_s}{N_r}\right) M$

$X_{lr}' = \omega_s L_{lr}'$



Assume: $i_{ar} = I_r \cos(s\omega_s t + \beta)$ $\quad i_{as} = I_s \cos(\omega_s t)$ $\quad \theta = (1-s)\omega_s t + \gamma$

$i_{br} = I_r \sin(s\omega_s t + \beta)$ $\quad i_{bs} = I_s \sin(\omega_s t)$

Voltage: $V_{as} = i_{as} R_s + \frac{d\lambda_{as}}{dt}$

$$\boxed{V_s \cos(\omega_s t + \theta_v) = R_s I_s \cos(\omega_s t) + \omega_s L_s I_s \cos(\omega_s t + 90^\circ) + \omega_s M I_r \cos(\omega_s t + (\gamma + \beta + 90^\circ))}$$

$V_{ar} = i_{ar} R_r + \frac{d\lambda_{ar}}{dt}$

$$\boxed{V_r \cos(s\omega_s t + \theta_r) = R_r I_r \cos(s\omega_s t + \beta) + s\omega_s L_r I_r \cos(s\omega_s t + (\beta + 90^\circ)) + s\omega_s M I_s \cos(s\omega_s t + (-\gamma + 90^\circ))}$$

As Phasors: $$\boxed{\frac{V_s}{\sqrt{2}} \angle \theta_v = R_s \left(\frac{I_s}{\sqrt{2}} \angle 0 \right) + j\omega_s L_s \left(\frac{I_s}{\sqrt{2}} \angle 0 \right) + j\omega_s M \left(\frac{I_r}{\sqrt{2}} \angle \gamma + \beta \right)}$$

$$\frac{V_r}{\sqrt{2}} \angle \theta_r = R_r \left(\frac{I_r}{\sqrt{2}} \angle \beta \right) + js\omega_s L_r \left(\frac{I_r}{\sqrt{2}} \angle \beta \right) + js\omega_s M \left(\frac{I_s}{\sqrt{2}} \angle -\gamma \right)$$

$$\rightarrow \boxed{\frac{V_r}{s\sqrt{2}} \angle \theta_r + \gamma = \frac{R_r}{s} \left(\frac{I_r}{\sqrt{2}} \angle \beta + \gamma \right) + j\omega_s L_r \left(\frac{I_r}{\sqrt{2}} \angle \beta + \gamma \right) + j\omega_s M \left(\frac{I_s}{\sqrt{2}} \angle 0 \right)}$$

Recall: $M = \frac{\mu_0 \pi r l N_s N_r}{2g}$

* use this to define

$L_s = L_{ls} + \left(\frac{N_s}{N_r} \right) M$

$L_r = L_{lr} + \left(\frac{N_r}{N_s} \right) M$

$$\boxed{\begin{aligned}\frac{V_s}{\sqrt{2}}\angle\theta_v &= R_s\left(\frac{I_s}{\sqrt{2}}\angle 0\right) + j\omega_s L_{\ell s}\left(\frac{I_s}{\sqrt{2}}\angle 0\right) + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_s}{\sqrt{2}}\angle 0\right)\\ &\quad + j\omega_s M\left(\frac{I_r}{\sqrt{2}}\angle\gamma+\beta\right)\end{aligned}}$$

$$\boxed{\begin{aligned}\frac{V_r}{s\sqrt{2}}\angle\theta_r+\gamma &= \frac{R_r}{s}\left(\frac{I_r}{\sqrt{2}}\angle\beta+\gamma\right) + j\omega_s L_{\ell r}\left(\frac{I_r}{\sqrt{2}}\angle\beta+\gamma\right) + j\omega_s M\left(\frac{N_r}{N_s}\right)\left(\frac{I_r}{\sqrt{2}}\angle\beta+\gamma\right)\\ &\quad + j\omega_s M\left(\frac{I_s}{\sqrt{2}}\angle 0\right)\end{aligned}}$$

Define Rotor Values referenced to the Stator:

$$I_r' = I_r\left(\frac{N_r}{N_s}\right) \qquad V_r' = \left(\frac{N_s}{N_r}\right)V_r$$

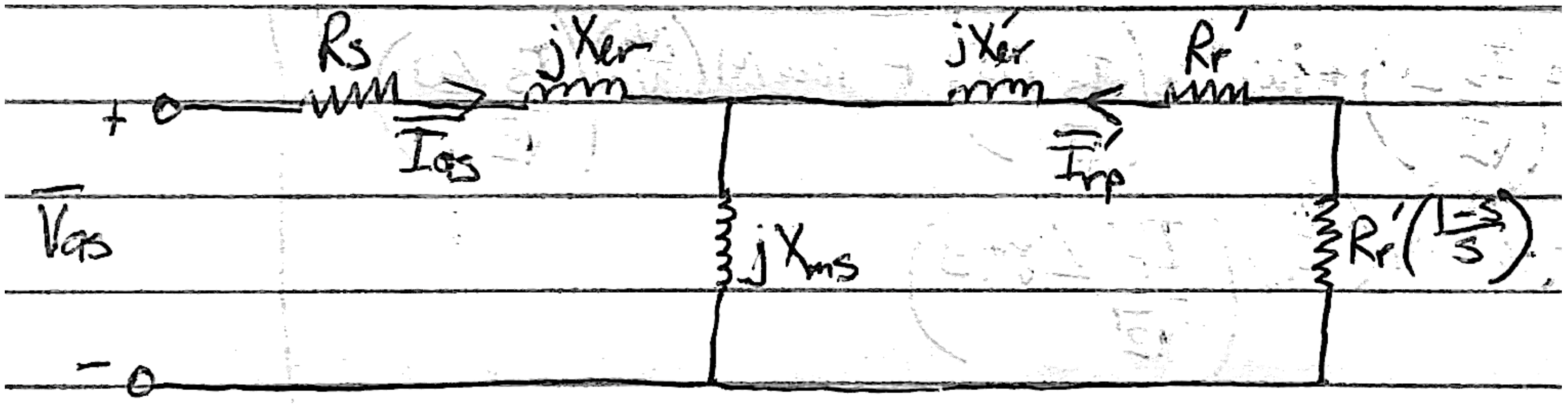
$$R_r' = R_r\left(\frac{N_s}{N_r}\right)^2 \qquad L_{\ell r}' = L_{\ell r}\left(\frac{N_s}{N_r}\right)^2$$

$$\boxed{\begin{aligned}\frac{V_s}{\sqrt{2}}\angle\theta_v &= R_s\left(\frac{I_s}{\sqrt{2}}\angle 0\right) + j\omega_s L_{\ell s}\left(\frac{I_s}{\sqrt{2}}\angle 0\right) + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_s}{\sqrt{2}}\angle 0\right)\\ &\quad + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_r'}{\sqrt{2}}\angle\gamma+\beta\right)\\[1em] \frac{V_r'}{s\sqrt{2}}\angle\theta_r+\gamma &= \frac{R_r'}{s}\left(\frac{I_r'}{\sqrt{2}}\angle\beta+\gamma\right) + j\omega_s L_{\ell r}'\left(\frac{I_r'}{\sqrt{2}}\angle\beta+\gamma\right)\\ &\quad + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_r'}{\sqrt{2}}\angle\beta+\gamma\right) + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_s}{\sqrt{2}}\angle 0\right)\end{aligned}}$$

Two types of Machines

- Wound Rotor: connect something to $\overline{V_{ar}'}$
- Squirrel cage: $\overline{V_{ar}'} = 0$
- For $\overline{V_{ar}'} = 0$

  $\frac{R_r'}{s} = R_r' + R_r'\left(\frac{1-s}{s}\right)$

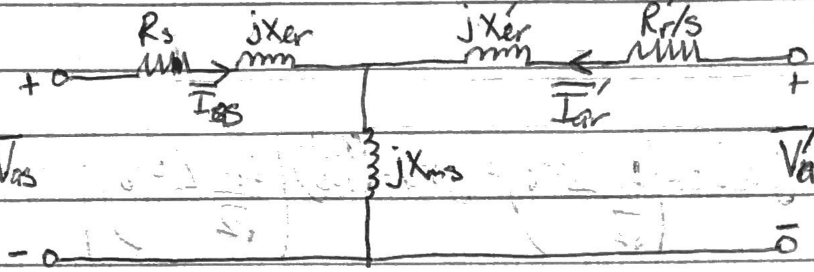


$* \ \overline{I_{ar}'} \neq \overline{I_{rp}'}$ (different angles)

As a circuit: $X_{\ell s} = \omega_s L_{\ell s}$

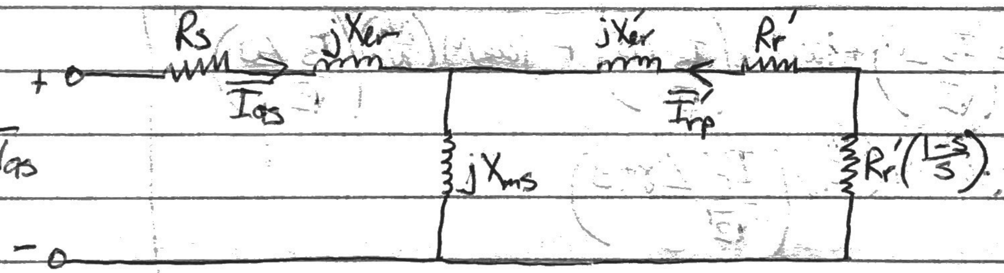
$X_{Ms} = \omega_s \left(\frac{N_s}{N_r}\right) M$

$X'_{\ell r} = \omega_s L'_{\ell r}$



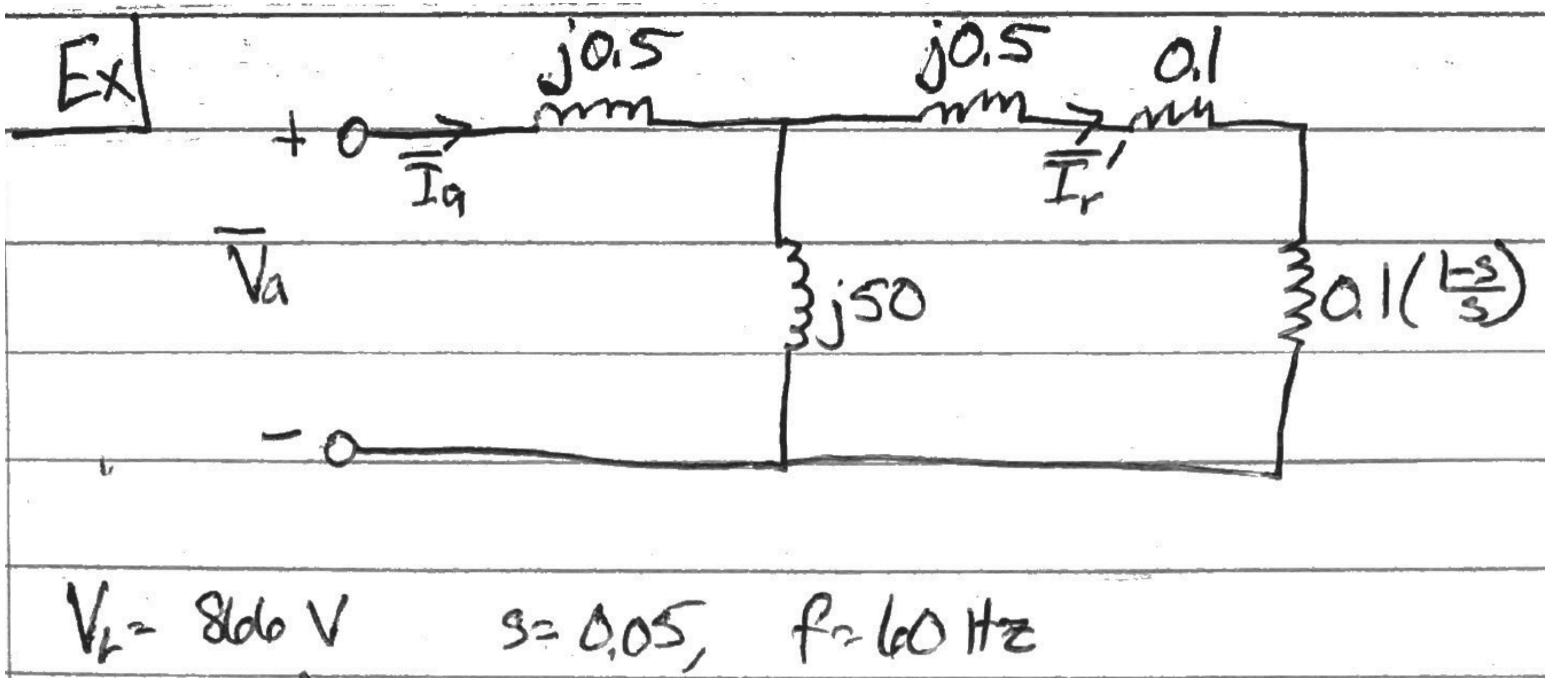
* Two types of machines:
  1) Wound Rotor: connect something to $\overline{V}'_{ar}$
  2) Squirrel cage: $\overline{V}'_{ar} = 0$

For $\overline{V}'_{ar} = 0$: $\frac{R'_r}{s} = R'_r + R'_r\left(\frac{1-s}{s}\right)$



* $\overline{I}'_{ar} \neq \overline{I}'_{rp}$ (different angles)

Example



What is the frequency of the rotor current?

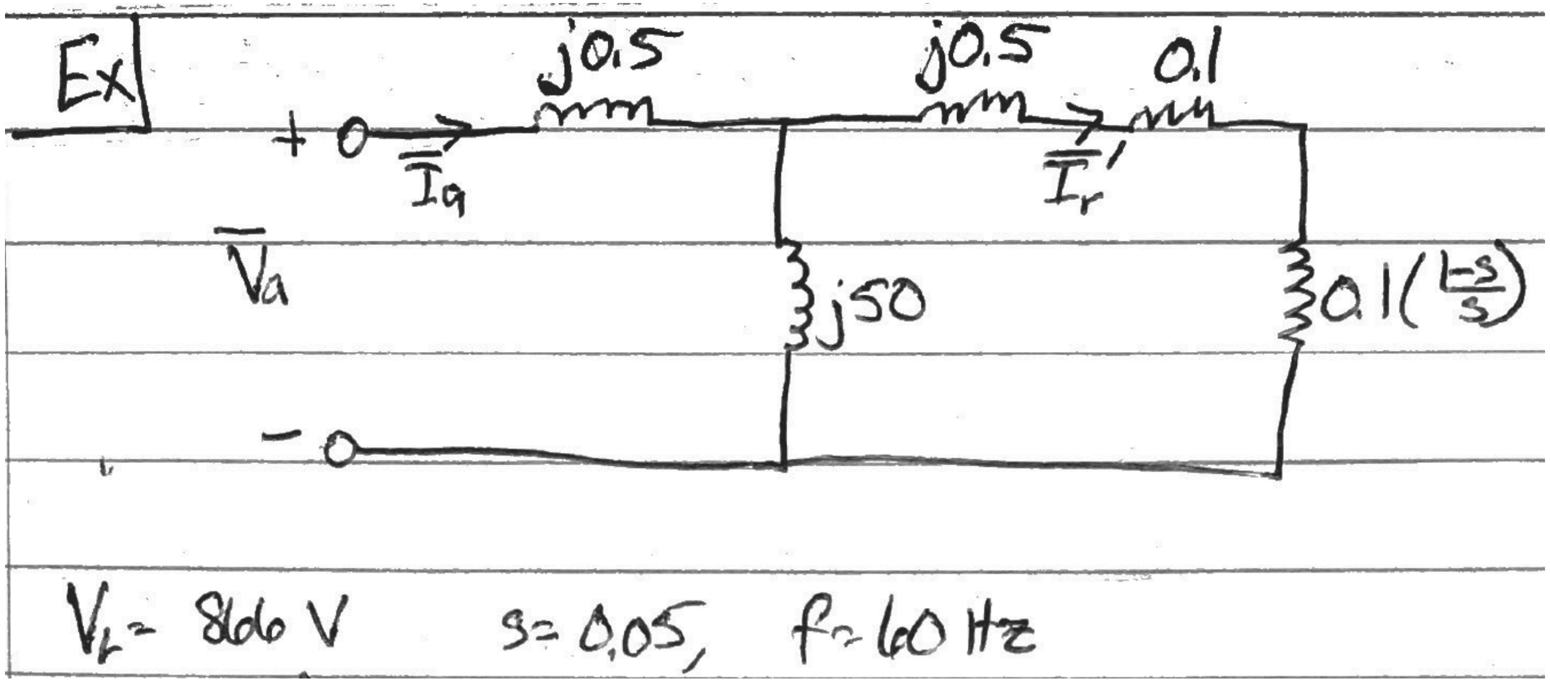
a) 57 Hz

b) 57 rad/s

c) 3 Hz

d) 3 rad/s

Example



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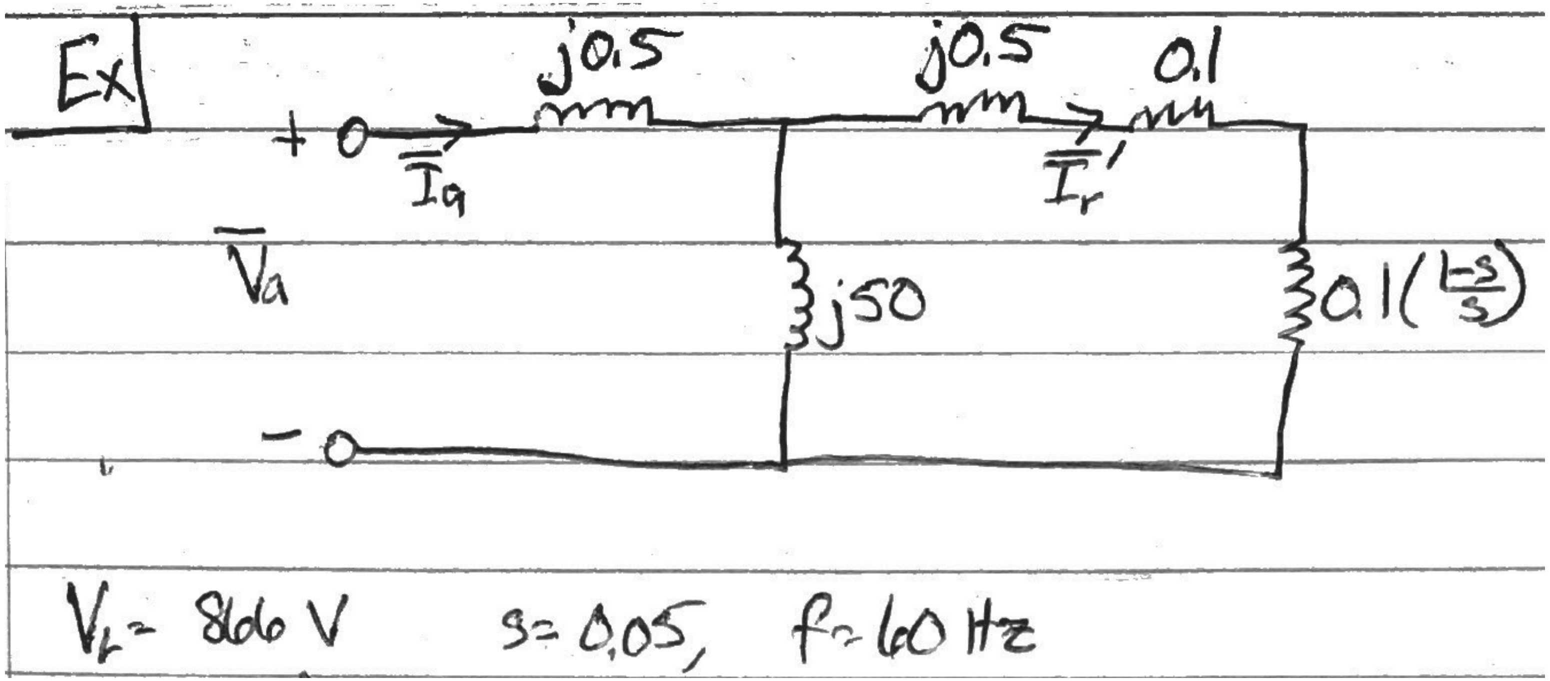


What is the 3-phase power supplied to the machine?

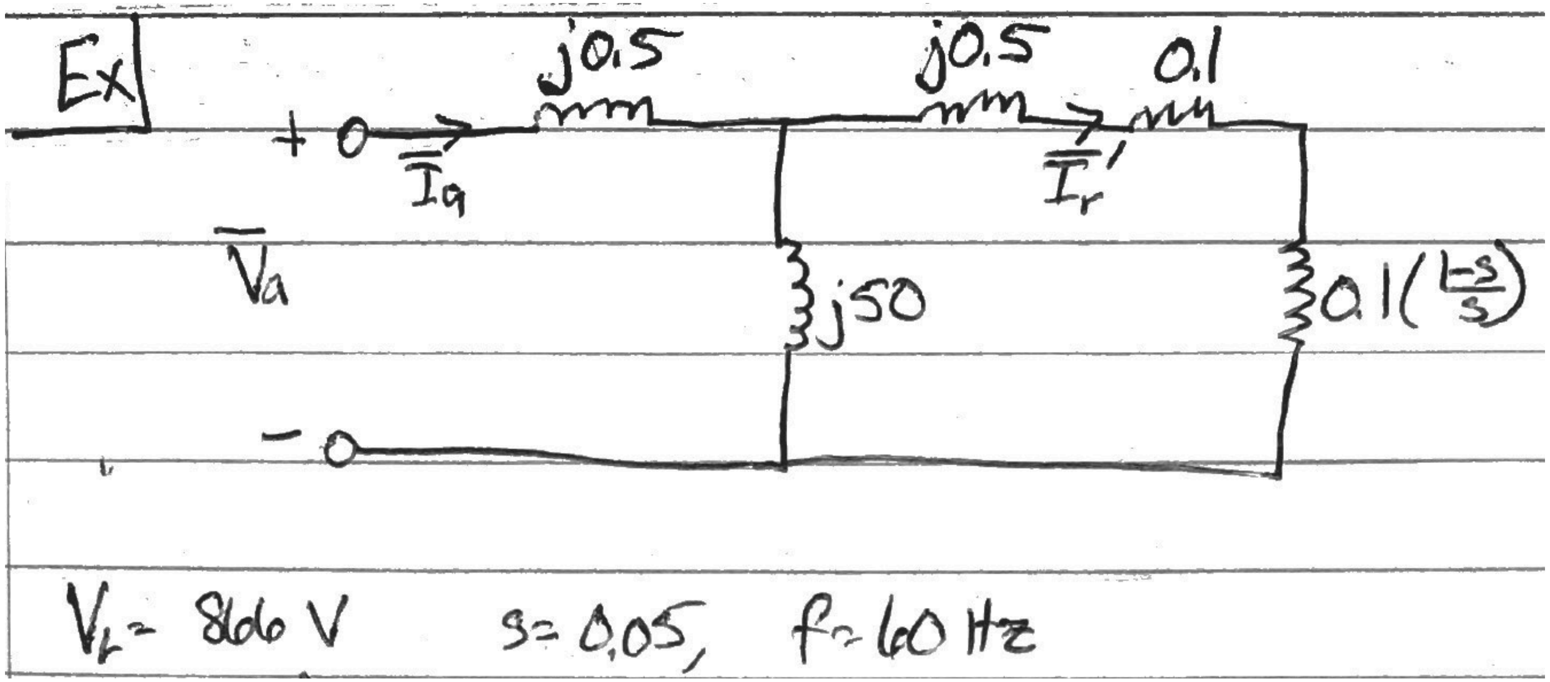
a) $336.02\angle 28.72°$ kVA

b) $336.02\angle -28.72°$ kVA

c) $194.0\angle 28.72°$ kVA

d) $194.0\angle -28.72°$ kVA

Example



What is the 3-phase power supplied to the machine?

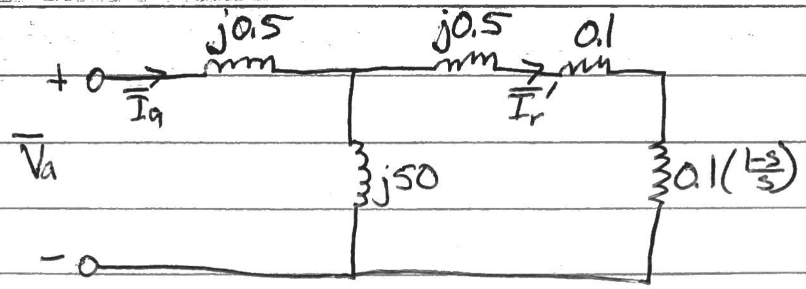
a) 336.02∠28.72° kVA

b) 336.02∠ − 28.72° kVA

c) 194.0∠28.72° kVA

d) 194.0∠ − 28.72° kVA

Ex

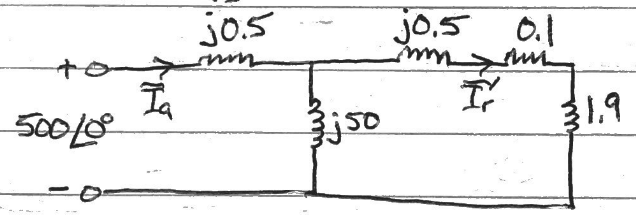


$V_L = 866\text{ V}$ $s = 0.05$, $f = 60\text{ Hz}$

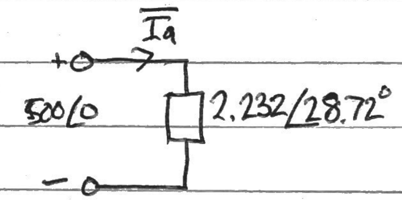
Find: a) $\overline{S}_{3\phi}$

b) $\overline{I}_r'$

Solution: $V_{an} = \frac{V_L}{\sqrt{3}} = 500\text{V}$



$\Rightarrow$



$\overline{I}_a = 224.014\angle -28.72^\circ\text{A}$

$\overline{S}_{3\phi} = 3\overline{V}_a\overline{I}_a^* \Rightarrow \overline{S}_{3\phi} = 3(500\angle 0^\circ)(224.014\angle 28.72^\circ)$

$$\boxed{\overline{S}_{3\phi} = 336.02\angle 28.72^\circ\text{ kVA}}$$

$\overline{V}_r' = 500\angle 0^\circ - j0.5\overline{I}_a \Rightarrow \overline{V}_r' = 446.177 - j98.2277 \Rightarrow \overline{V}_r' = 456.862\angle -12.416^\circ\text{V}$

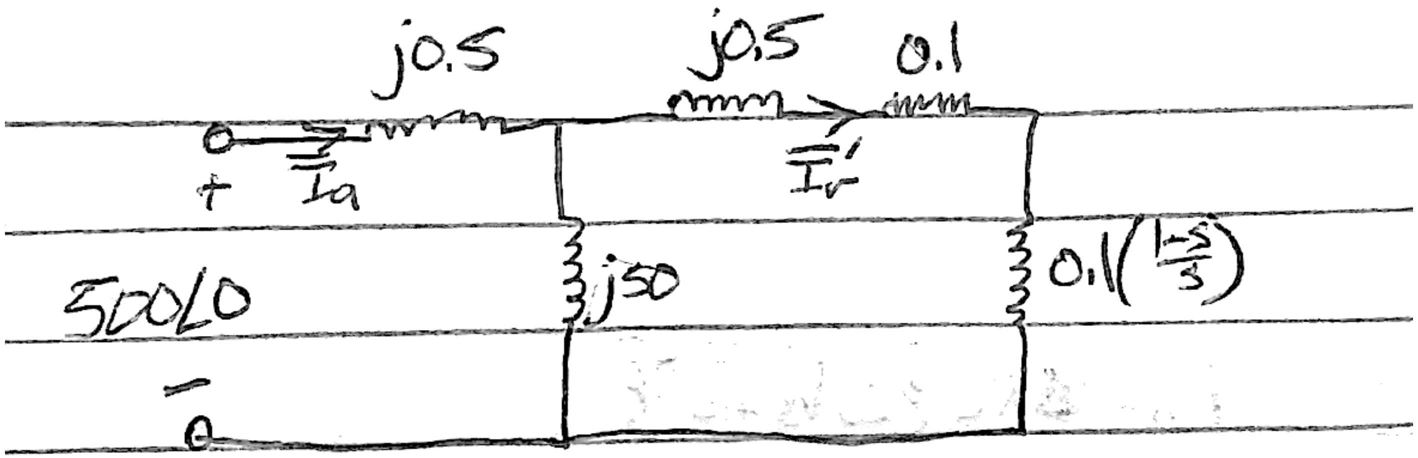
$\overline{V}_r' = (2 + j0.5)\overline{I}_r'$

$$\boxed{\overline{I}_r' = 221.606\angle -26.452^\circ\text{A}}$$

Definitions

- Input real power: $P_{3\phi} = Re\{3\overline{V_{an}}\,\overline{I_{as}}^*\}$
- Stator copper loss: $P_{SCL} = 3R_s|\overline{I_s}|^2$
- Rotor copper loss: $P_{RCL} = 3R'_r\left|\overline{I'_r}\right|^2$
- Power across the gap: $P_{ag} = 3\,\frac{R'_r}{s}\left|\overline{I'_r}\right|^2$
- Mechanical Power: $P_m = (1 - s)P_{ag}$
- Multipole system: $\omega_m = (1 - s)\omega_s\left(\frac{2}{p}\right)$
- Torque: $T^e = \frac{(1-s)P_{ag}}{(1-s)\omega_s\left(\frac{2}{p}\right)} = \frac{P_{ag}}{\omega_s\left(\frac{2}{p}\right)}$

Example



$s = 0.05$ $f = 60\text{Hz}$ 2 pole

from last time: $\bar{I}_a = 224.014\angle -28.74^\circ$ A

$\bar{I}_r' = 221.606\angle -26.452^\circ$ A

What is the torque?

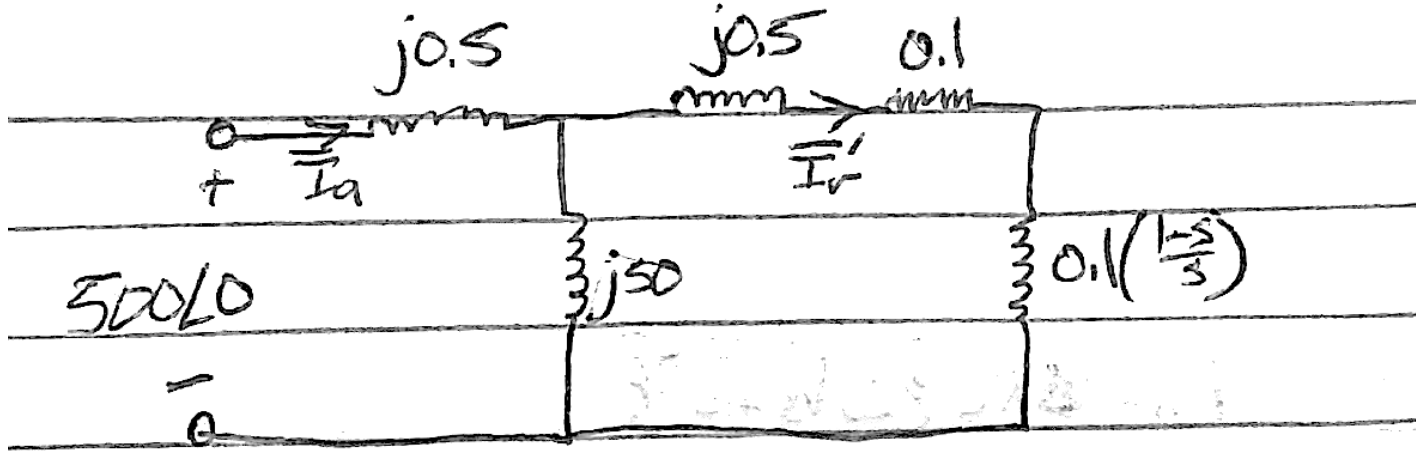
a) 822.7 Nm

b) 742.5 Nm

c) 781.6 Nm

d) 4910.9 Nm

Example



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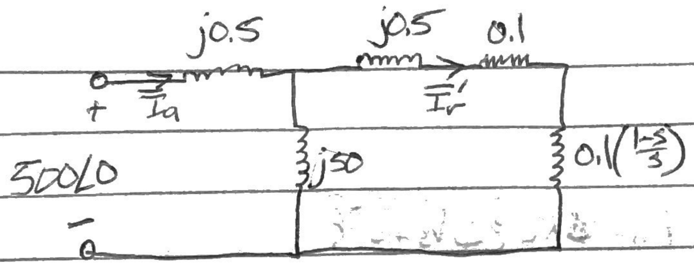

$s = 0.05 \qquad f = 60\text{Hz} \qquad 2 \text{ pole}$

from last time: $\bar{I}_a = 224.014\angle -28.74°\ A$

$\bar{I}_r' = 221.606\angle -26.452°\ A$

Find: T^e

$P_{AG} = 3|\bar{I}_r'|^2\left(\frac{R_r'}{s}\right) \Rightarrow P_{AG} = 3(221.606)^2(2) = 294.655\ kW$

$P_m = (1-s)P_{AG} \Rightarrow P_m = 279.923\ kW$

$T^e = \frac{P_m}{(1-s)\omega_s} \Rightarrow \boxed{T^e = 781.597\ Nm}$